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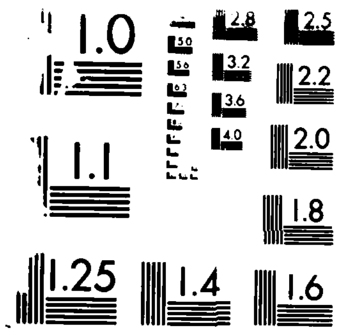


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# PART VII: ORBITS OF HIGH ECCENTRICITY, WITH SCALE HEIGHT DEPENDENT ON ALTITUDE

by


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# THE CONTRACTION OF SATELLITE ORBITS UNDER THE INFLUENCE OF AIR DRAG

## PART VII: ORBITS OF HIGH ECCENTRICITY, WITH SCALE HEIGHT DEPENDENT ON ALTITUDE

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SUMMARY

Part III of this series of papers developed the theory of high-eccentricity orbits ($e > 0.2$) in an atmosphere having an exponential variation of air density with height, that is, with the density scale height $H$ taken as constant. Part IV derived the appropriate theory for low-eccentricity orbits ($e < 0.2$) in a more realistic atmosphere where $H$ varies linearly with height $y$ (and $\mu = dH/dy < 0.2$). The present Report treats the orbits of Part III when they meet the air drag specified by the atmospheric model of Part IV. Equations are derived showing how the perigee height varies with eccentricity, and the eccentricity varies with time, over the major part of the satellite's life. It is shown that the theory of Part III remains valid, to order $\mu^2$, if $H$ is evaluated at a specific height above perigee.

Departmental Reference: Space 656

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## 1 INTRODUCTION

Part I of this series of papers[1] gave the theory for the contraction of Earth satellite orbits of eccentricity $e < 0.2$ under the influence of air drag in a spherically symmetrical atmosphere, over a single revolution and also over the complete lifetime. Part II did the same for an oblate atmosphere[2]. In Part III the theory of Part I was extended[3] to orbits of high eccentricity, $0.2 \leqslant e < 1$. In Parts I to III the air density was assumed to vary exponentially with height, having a constant density scale height $H$: in Part IV the theory was developed[4], for $e < 0.2$, on the assumption that instead of being constant, $H$ was a linear function of height $y$. Parts V and VI took account of the day-to-night variation in air density[5,6], again for $e < 0.2$, and reverting to constant $H$; Part VI was devoted to the special conditions for near-circular orbits. Subsequently the theory has been extended by Swinerd and Boulton[7,8] to cover the combined effects of oblateness, day-to-night variation and variation of $H$ with height, for $e < 0.2$, over a single revolution.

The present Report can be looked on as a marriage between Part III and Part IV, and applies for orbits of high eccentricity $(0.2 \leqslant e < 1)$ in an atmosphere where $H$ varies linearly with height. The theory is required for three reasons:

(a) In the real atmosphere $H$ is not constant, but does vary almost linearly with height; so the constant-H theory of Part III may be inadequate in practice, and the extent of the inadequacies is unknown until the new theory has been developed.

(b) The theory is needed in interpreting the measurements of geophysical parameters from orbital changes for orbits with eccentricity greater than 0.2: without it, the height at which the measurements apply remains uncertain.

(c) In orbit determination programs, such as PROP[9], a constant-H theory is used in the model, and the new theory will allow the best constant value of $H$ to be selected for a particular orbit determination.

## 2 ASSUMPTIONS

The theory is developed under the nine assumptions listed below. These assumptions are the same as in Part I, except for (c) and (h) and a minor modification in (e).

(a) The atmosphere is spherically symmetrical.

(b) The air density $\rho$ at a given distance from the Earth's centre does not vary with time.

(c) The air density $\rho$ and the quantity $-\rho/(d\rho/dr)$, which will be called the *density scale height* and denoted by $H$, vary with distance $r$ from the Earth's centre in the manner specified in section 3.

(d) The resultant aerodynamic force on the satellite acts in the direction opposite to the velocity $V$ of the satellite relative to the ambient air, and may be taken as $\frac{1}{2}\rho V^2 SC_D$, where $C_D$ is a drag coefficient based on an effective cross-sectional area $S$, and $SC_D$ is assumed constant.

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(e) The atmosphere rotates at a constant angular velocity $w$ not very different from that of the Earth.

(f) The Earth's gravitational potential is taken as that of a point mass at its centre, so that the unperturbed orbit is an ellipse in a plane passing through the Earth's centre.

(g) During one revolution the action of air drag changes the orbit by only a small amount, whose square can be neglected.

(h) The orbital eccentricity $e$ lies within the limits $0.2 \leqslant e < 1$ .

(i) Lunisolar perturbations are ignored.

The assumptions have been discussed in Part I, so little more needs to be said here, but three points do deserve mention. First, high-eccentricity orbits are not much affected by atmospheric oblateness, so that assumption (a) is not objectionable. Second, the neglect of the asymmetrical components of the Earth's gravitational field, particularly the third harmonic, is questionable and it is intended to develop methods for taking account of this effect in a subsequent paper. (For a spherically symmetrical atmosphere the effects of the much larger second harmonic are not important.) The third, and most important, of the simplifications is the neglect of long-term lunisolar perturbations, although their effect on the single-revolution results is negligible. The problem arises over months or years, when lunisolar perturbations can be very large for high-eccentricity orbits. However, the effects are then to some extent self-eliminating, because the perigee will either rise to heights where the drag is obviously negligible, or will be forced down to a height where the satellite will decay. Even in the long term, lunisolar perturbations to the perigee height are generally small for orbits of eccentricity less than about 0.5, and the theory is often also still applicable at higher eccentricities. At eccentricities higher than 0.8, however, lunisolar perturbations are generally dominant and we shall regard 0.8 as the likely practical upper limit for $e$ .

## 3 AIR DENSITY

The model for the variation of density $\rho$ with distance $r$ from the Earth's centre is as specified in Part IV. The density scale height $H$ is defined by the equation

$$H = -\frac{\rho}{d\rho/dr} , \tag{1}$$

and if $H$ were constant equation (1) could be integrated to give the simple exponential variation of density with height,

$$\rho = \rho_p \exp\left(-\frac{r - r_p}{H}\right) , \tag{2}$$

where suffix $p$ denotes conditions at perigee, and in practice equation (2) is used only for $r > r_p$ .

Here we assume instead that

$$\rho = \rho_p\left\{1 + b\left(r - r_p\right)^2\right\} \exp\left(-\frac{r - r_p}{H_p}\right) , \tag{3}$$

where $H_p$ and $b$ are taken constant during one revolution. On substituting (3) into (1), and writing $\alpha = 2bH_p^2$ and $\left(r - r_p\right)/H_p = \eta$ , we find

$$H = H_p + \frac{\alpha\left(r - r_p\right)}{1 - \alpha\left(\eta - \frac{1}{2}\eta^2\right)} = H_p + \mu\left(r - r_p\right) , \qquad \text{say,} \tag{4}$$

where
$$\alpha = 2bH_p^2 = \frac{\mu}{1 + \mu\left(\eta - \frac{1}{2}\eta^2\right)} . \tag{5}$$

At heights less than 2.4 $H_p$ above perigee, *ie* for $\eta < 2.4$ , the term $\left(\eta - \frac{1}{2}\eta^2\right) \leqslant \frac{1}{2}$ , and if $\mu$ is small it will usually be an adequate approximation to take $\alpha = \mu$ . (The situation for $\eta > 2.4$ is discussed in section 5.)

In practice $\mu$ is usually of order 0.1: Fig 1 shows the variation of $H$ and of $\mu$ given by[10] the *COSPAR International Reference Atmosphere 1972*, for two levels of solar activity. It is seen that a linear variation of $H$ over a height range of 50-100 km is a satisfactory approximation and that $\mu$ does not exceed 0.2 unless the height falls below 190 km (low solar activity) or 230 km (high solar activity). In developing the theory in this Report we shall assume $\mu < 0.2$ .

It may seem simplest to evaluate $\mu$ at perigee height. But the subsequent analysis shows that the value of $H$ at a height 1.5 $H_p$ above perigee is of special importance; so it is recommended that the numerical value of $\mu$ at a height about 1.5 $H_p$ above perigee should be used. With this rule, Fig 1 shows that $\mu < 0.2$ if the perigee height exceeds 160 km (low solar activity) or 190 km (high solar activity).

The expression (3) for $\rho$ will be used for integrations over one revolution, to obtain the changes in the orbital elements over one revolution. These equations are then integrated over a long time-interval, perhaps nearly all the satellite's lifetime. During this time there will be a variation in the perigee height and therefore in $H_p$ . For this further integration we assume, in accordance with (4), that

$$H_p = H_{p0} + \mu\left(r_p - r_{p0}\right) , \tag{6}$$

where the second suffix 0 denotes initial values. If $0.2 \leqslant e \leqslant 0.8$ , it is found that $0 \leqslant r_{p0} - r_p \leqslant 0.5\ H_{p0}$ . Thus the change in $H_p$ due to the $\mu$ term is never more than 10% if $\mu < 0.2$ , and the error of order $\frac{1}{2}\mu$ between $b$ and $\mu$ in equation (5) leads to an error of less than 1% in $H_p$ . This is acceptable, because $H_p$ is rarely known with an accuracy better than 2%. We shall therefore normally utilize equation (5) in the form $b = \mu/2H_p^2$ , although the fuller version is needed at one point in the analysis.

# 4 BASIC EQUATIONS

## 4.1 Air density

If $a$ is the semi-major axis of the orbit, the radial distance $r$ of a satellite may be expressed in terms of the eccentric anomaly $E$ as

$$r = a(1 - e\cos E) \quad , \tag{7}$$

so that

$$r - r_p = ae(1 - \cos E) \quad , \tag{8}$$

and (3) becomes

$$\rho = \rho_p\left\{1 + bx^2(1 - \cos E)^2\right\}\exp\left\{-z(1 - \cos E)\right\} \quad , \tag{9}$$

where

$$x = ae \tag{10}$$

and

$$z = \frac{ae}{H_p} \quad . \tag{11}$$

A typical value of $a/H_p$ is 150, and so values of $e$ greater than 0.2 correspond to values of $z$ greater than about 30. Equation (9) specifies $\rho$ in terms of $E$ .

## 4.2 Changes in $a$ and $x$

The changes in $a$ and $x$ per revolution, denoted by $\Delta a$ and $\Delta x$ respectively, are given by equations (12) and (13) of Part I as

$$\Delta a = -2a^2\delta\int_0^{\pi}\frac{(1 + e\cos E)^{\frac{3}{2}}}{(1 - e\cos E)^{\frac{1}{2}}}\rho dE \tag{12}$$

$$\Delta x = -2a^2\delta\int_0^{\pi}\left(\frac{1 + e\cos E}{1 - e\cos E}\right)^{\frac{1}{2}}(\cos E + e)\rho dE \quad , \tag{13}$$

on replacing the integrals from 0 to $2\pi$ by twice the integrals from 0 to $\pi$ . Here $\delta$ is a constant given by $\delta = FSC_D/m$ , where $m$ is the mass of the satellite and $F$ is a factor which allows for the rotation of the atmosphere. From Part I,

$$F = \left(1 - \frac{r_{p0}w}{v_{p0}}\cos i\right)^2 \quad , \tag{14}$$

where $v$ is the velocity of the satellite relative to the Earth's centre and $i$ the inclination of the orbit to the equator.

# 5 EVALUATION OF $\Delta a$ and $\Delta x$

When $e > 0.2$ , the integration of (12) and (13) can best be performed by expressing $E$ in terms of $\lambda$ , where

$$\cos E = 1 - \lambda^2/z \quad , \tag{15}$$

so that

$$\sin E = \lambda\left(\frac{2}{z}\right)^{\frac{1}{2}}\left(1 - \frac{\lambda^2}{2z}\right)^{\frac{1}{2}} \tag{16}$$

and

$$dE = \left(\frac{2}{z}\right)^{\frac{1}{2}}\left(1 - \frac{\lambda^2}{2z}\right)^{-\frac{1}{2}} d\lambda \quad . \tag{17}$$

Substituting (15) into (9) gives

$$\rho = \rho_p\left\{1 + \frac{bx^2\lambda^4}{z^2}\right\}\exp\left(-\lambda^2\right) \quad , \tag{18}$$

which may be expressed in terms of $\mu$ by using equation (5), as

$$\rho = \rho_p\left[1 + \tfrac{1}{2}\mu\lambda^4\left\{1 - \tfrac{1}{2}\mu\lambda^2\left(\lambda^2 - 2\right)\right\}^{-1}\right]\exp\left(-\lambda^2\right) \quad . \tag{19}$$

We now make the simplifying assumption that the terms of order $\mu^2$ in (19) can be ignored, and take as our standard equation for density

$$\rho = \rho_p\left(1 + \tfrac{1}{2}\mu\lambda^4\right)\exp\left(-\lambda^2\right) \quad . \tag{20}$$

From (19) the error in equation (20) is $\frac{1}{4}\rho_p\mu^2\lambda^6\left(\lambda^2 - 2\right)\exp\left(-\lambda^2\right)$ . For heights up to 2.4 $H_p$ above perigee $\left(\lambda^2 < 2.4\right)$ this is less than $0.125\mu^2\rho_p$, so that the error in (20) is less than $0.005\rho_p$ for $\mu < 0.2$ . At heights more than 2.4 $H_p$ above perigee, this error can be larger, having a maximum of $0.025\rho_p$ at 4.7 $H_p$ above perigee for $\mu = 0.2$ : at such great heights, however, the assumption of constant $\mu$ has itself become invalid, and it is pointless to pursue the effect of $0\left(\mu^2\right)$ terms into these rarefied realms. To summarize, we are seeking a good approximation to the density in the region from perigee up to about 2.4 $H_p$ above, without being too much concerned about greater heights, where the error could increase to $0.6\mu^2\rho_p$, if $\mu$ remained constant.

If we substitute for $\cos E$ by (15), $dE$ by (17) and $\rho$ by (20), we find that equations (12) and (13) can be expanded in powers of $\lambda^2/z$ to give

$$\Delta a = -2a^2\rho_p\delta\left(\frac{2}{z}\right)^{\frac{1}{2}}\frac{(1+e)^{\frac{3}{2}}}{(1-e)^{\frac{1}{2}}}\int_0^{\sqrt{2z}}\left\{1 - \frac{\left(8e - 3e^2 - 1\right)\lambda^2}{4z\left(1-e^2\right)} + \frac{K_1\lambda^4}{z^2} + 0\left(\frac{\lambda^6}{z^3}\right)\right\}\left(1 + \tfrac{1}{2}\mu\lambda^4\right)\exp\left(-\lambda^2\right)d\lambda$$

...... (21)

$$\Delta x = -2a^2\rho_p\delta\left(\frac{2}{z}\right)^{\frac{1}{2}}\frac{(1+e)^{\frac{3}{2}}}{(1-e)^{\frac{1}{2}}}\int_0^{\sqrt{2z}}\left\{1 - \frac{\left(3 + e^2\right)\lambda^2}{4z\left(1-e^2\right)} + \frac{K_2\lambda^4}{z^2} + 0\left(\frac{\lambda^6}{z^3}\right)\right\}\left(1 + \tfrac{1}{2}\mu\lambda^4\right)\exp\left(-\lambda^2\right)d\lambda \quad ,$$

...... (22)

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where $K_1$ and $K_2$ are functions of $e$ specified in Part III. Since $\sqrt{2z} > 6$ for $e > 0.2$, the limit $\sqrt{2z}$ in (21) and (22) may be replaced by $\infty$, with relative error $<10^{-16}$. Also if we write

$$\int_0^\infty \lambda^{2n} \exp\left(-\lambda^2\right) d\lambda = G_n \quad , \tag{23}$$

it is easily shown that $G_n = \frac{1}{2}(2n-1)G_{n-1}$ for $n \geqslant 1$, and, since $G_0 = \frac{1}{2}\sqrt{\pi}$, we have

$$G_0 = \frac{1}{2}\sqrt{\pi}, \qquad G_1 = \frac{1}{4}\sqrt{\pi}, \qquad G_2 = \frac{3}{8}\sqrt{\pi}, \qquad G_3 = \frac{15}{16}\sqrt{\pi}, \qquad G_4 = \frac{105}{32}\sqrt{\pi} \quad . \tag{24}$$

With the aid of (24), equations (21) and (22) may be integrated to give:

$$\Delta a = -a^2 \rho_p \delta\left(\frac{2\pi}{z}\right)^{\frac{1}{2}} \frac{(1+e)^{\frac{3}{2}}}{(1-e)^{\frac{1}{2}}} \left\{1 - \frac{8e - 3e^2 - 1}{8z\left(1-e^2\right)} + \frac{3}{8}\mu - \frac{15\mu\left(8e - 3e^2 - 1\right)}{64z\left(1-e^2\right)} + 0\left(\frac{2}{z^3}\right)\right\} \tag{25}$$

$$\Delta x = -a^2 \rho_p \delta\left(\frac{2\pi}{z}\right)^{\frac{1}{2}} \frac{(1+e)^{\frac{3}{2}}}{(1-e)^{\frac{1}{2}}} \left\{1 - \frac{3 + e^2}{8z\left(1-e^2\right)} + \frac{3}{8}\mu - \frac{15\mu\left(3 + e^2\right)}{64z\left(1-e^2\right)} + 0\left(\frac{2}{z^3}\right)\right\} \quad . \tag{26}$$

Here two terms inside the curly brackets, $3K_1/4z^2$ in (25) and $3K_2/4z^2$ in (26), have been omitted because it was found in Part III that they are less than 0.00005 if $H/r_p = 0.008$, for $0.2 \leqslant e < 1$.

## 6 PERIGEE DISTANCE AS A FUNCTION OF e

In Part III it was shown that when equation (25) with $\mu = 0$ is divided by equation (26) with $\mu = 0$, the $0\left(1/z^2\right)$ terms are less than $0.15/z^2$, that is, less than 0.00016. Hence

$$\frac{\Delta a}{\Delta x} = 1 + \frac{1-e}{2z(1+e)} + \frac{3\mu(1-e)}{4z(1+e)} + 0\left(\frac{0.35\mu^2}{z}, \frac{0.15}{z^2}\right) \quad , \tag{27}$$

where the $0\left(0.15/z^2\right)$ term is taken as incorporating a term $0\left(\mu/z^2\right)$, and $0.35\mu^2/z$ is the maximum value of this term (when $e = 0.2$ and $z = 30$). If we write

$$H_1 = H_p\left(1 + \frac{3}{2}\mu\right) \quad , \tag{28}$$

so that, as (4) shows, $H_1$ is the value of $H$ at a height $\frac{3}{2}H_p$ above perigee, equation (27) reduces to

$$\frac{da}{dx} = 1 + \frac{1-e}{2(1+e)}\left(\frac{H_1}{x}\right) + 0\left(\frac{0.35\mu^2}{z}, \frac{0.15}{z^2}\right) \quad . \tag{29}$$

Thus, as with eccentricities between about 0.03 and 0.2, the constant-H equation for da/dx may be used if H is evaluated at a height $\frac{3}{2}$ $H_p$ above perigee. Conversely, if an orbit is accurately determined from observations and accurate values of da/dx, e and x are available, equation (29) can be used to evaluate H at a height $\frac{3}{2}$ $H_p$ above perigee.

This is a neat and useful one-revolution result; but, in order to integrate (29) over the whole lifetime of the satellite, we have to take account of the fact that $H_1$ is evaluated at a height, $y_1$ say; which is slowly decreasing as $r_p$ decreases. We need a constant 'mean' value for $H_1$ and fortunately an appropriate value can easily be guessed. If we assume that $e_0 = 0.8$ is the highest practicable value of e , the maximum decrease in perigee height is, from Fig 3 of Part III, $0.5H \simeq 0.5H_{p0}$ (when $e_0 = 0.8$ and $e = 0.2$ ). As the orbit spends more time at high eccentricity than at low (see Fig 8 of Part III), the mean perigee height is likely to be about $0.2H_{p0}$ below the initial perigee when the orbit is contracting from $e_0 = 0.8$ to $e = 0.2$ . At the other extreme, when the orbit contracts very little, the mean perigee height is virtually equal to the initial perigee height. Thus the mean decrease in perigee height during the life, for all varieties of orbit, is likely to be between 0 and $0.2\ H_{p0}$ and may be taken as $0.1\ H_{p0}$ with maximum error $0.1\ H_{p0}$ . Hence the most promising mean height for evaluating H is at a height $1.4\ H_{p0}$ above the initial perigee. So we define

$$H^* = H_{p0}(1 + 1.4\mu) \quad , \tag{30}$$

and try it as a mean value for $H_1$ .

Dividing (28) by (30), we find

$$H_1 = H^*\left\{1 + \mu(0.1 - \sigma) - \mu^2(1.5\sigma + 0.14) + 0\left(\mu^3\right)\right\} \quad , \tag{31}$$

where

$$\sigma = \frac{r_{p0} - r_p}{H^*} \quad . \tag{32}$$

The variation of $\sigma$ with $e/e_0$ as given by Part III is shown in Fig 2: we need a simple approximation for $\sigma$ in terms of e and we take

$$\sigma = \frac{r_{p0} - r_p}{H^*} = \left(0.4 - 0.3e_0\right)\left(\frac{e_0}{e} - 1\right) \pm 0.065 \quad , \tag{33}$$

which is shown by the broken lines in Fig 2. The error quoted is the maximum for $0.2 < e_0 < 0.8$ , and occurs when $e_0 = 0.8$ and $e/e_0 = 0.4$ . In view of (31) and (33), equation (29) may be written

$$\frac{da}{dx} = 1 + \frac{1 - e}{2(1 + e)} \frac{H^*}{x}\left[1 + \mu\left\{0.1 - \left(0.4 - 0.3e_0\right)\left(\frac{e_0}{e} - 1\right)\right\}\right] + O\left(\frac{0.35\mu^2}{z}, \frac{0.15}{z^2}\right) \qquad \ldots\ldots (34)$$

as the $\mu^2$ term in (31) and the error term in (33) lead to O-terms smaller than $O\left(0.35\mu^2/z\right)$. In (34), we need to express $e$ in terms of $x$. Since the perigee distance decreases by less than $\frac{1}{2}H^*$,

$$a = Q_0 + x + O(\tfrac{1}{2}H^*) , \qquad (35)$$

where $Q_0$ denotes the initial perigee distance,

$$Q_0 = a_0\left(1 - e_0\right) = a_0 - x_0 . \qquad (36)$$

From (35),

$$\frac{1}{e} = \frac{a}{x} = \frac{Q_0}{x} + 1 + O\left(\frac{1}{2z}\right) . \qquad (37)$$

Eliminating $e$ from (34) with the aid of (37), we find

$$\frac{1}{H^*}\frac{d(a - x)}{dx} = \frac{\left[1 + \mu\left\{0.1 + \left(0.4 - 0.3e_0\right)\left(1 - e_0\right)\right\}\right]Q_0}{2\left(Q_0 + 2x\right)x} - \frac{\mu e_0 Q_0^2\left(0.4 - 0.3e_0\right)}{2\left(Q_0 + 2x\right)x^2} + O\left(\frac{0.35\mu^2}{x}, \frac{0.15H}{x^2}\right). \qquad \ldots\ldots (38)$$

Integrating (38) and collecting terms, we have

$$\frac{r_{p0} - r_p}{H^*} = \tfrac{1}{2}\ln\frac{x_0\left(Q_0 + 2x\right)}{x\left(Q_0 + 2x_0\right)} + \mu\left[\left(0.5 + 0.1e_0 - 0.3e_o^2\right)\left\{\tfrac{1}{2}\ln\frac{x_0\left(Q_0 + 2x\right)}{x\left(Q_0 + 2x_0\right)}\right\} - \tfrac{1}{2}e_0\left(0.4 - 0.3e_0\right)\left(\frac{1 - e}{e} - \frac{1 - e_0}{e_0}\right)\right] + O\left(0.35\mu^2\ln\frac{x_0}{x}, \frac{0.15}{z}\right). \qquad \ldots\ldots (39)$$

The term in curly brackets may be written as $\sigma + O(\mu)$, which in turn may be expressed in the form (32), giving finally

$$\frac{r_{p0} - r_p}{H^*} = \tfrac{1}{2}\ln\left\{\frac{x_0\left(1 - e_0\right)}{x\left(1 + e_0\right)} + \frac{2e_0}{1 + e_0}\right\} - \frac{\mu e_0}{100}\left(4 - 3e_0\right)\left(3e_0 - 1\right)\left(\frac{e_0}{e} - 1\right) + 0\left(\mu^2 , \frac{0.15}{z}\right) ,$$

...... (40)

where the $0\left(\mu^2\right)$ term appears because $\ln\left(x_0/x\right)$ can be as large as 2.77 (when $e_0 = 0.8$ and $e = 0.2$ ) and $2.77 \times 0.35 = 0.97$ . The coefficient of $\mu$ in equation (40) has a maximum value, when $e = 0.2$ , of $0.01e_0\ (4 - 3e_0)(3e_0 - 1)(5e_0 - 1)$ , which is always less than 0.054 if $0.2 < e_0 < 0.8$ , and is less than 0.022 for $0.2 < e_0 < 0.6$ . The $\mu$ term in (40) is thus of order $0.05\mu = 0.25\mu^2$ if $\mu = 0.2$ , and can be absorbed in the $0\left(\mu^2\right)$ term.

Equation (40), shorn of its $\mu$ term, shows that the zero-$\mu$ theory can still be used, with error $0\left(\mu^2\right)$ , if $H$ is evaluated at a height $1.4\ H_{p0}$ above the initial perigee height.

Note also that, from equation (23) of Part III, equation (40) may be rewritten as

$$\frac{r_{p0} - r_p}{H^*} = \tfrac{1}{2}\ln\frac{e_0(1 + e)}{e\left(1 + e_0\right)} + 0\left(\frac{H}{4r_p} , \mu^2 , \frac{0.15}{z}\right) . \tag{41}$$

This form is appropriate because $H/4r_p$ ($\simeq 0.002$) is normally less than $\mu^2$ .

## 7 VARIATION OF e WITH t

From (25) and (26), we have

$$a\Delta e = \Delta x - e\Delta a = - a^2\rho_p\delta\left(\frac{2\pi H_p}{ae}\right)^{\frac{1}{2}} (1 + e)^{\frac{3}{2}}(1 - e)^{\frac{1}{2}}\left[1 - \frac{3 + 4e - 3e^2}{8z\left(1 - e^2\right)} + \frac{3}{8}\mu + 0\left(\frac{\mu}{z} , \frac{1}{z^2}\right)\right] ,$$

...... (42)

The $1/z$ term in (42) has the value $(0.34 \pm 0.04)/\left\{z(1 - e)\right\}$ for $0.2 < e < 0.8$ , which is of order 0.01 or less for $z(1 - e) > 30$ . The term is dropped after integration in Part III, and here it is more convenient to drop it at once. On dividing by

$$\Delta t = T = T_0\left(a/a_0\right)^{\frac{3}{2}} , \tag{43}$$

where $T$ is the orbital period, equation (42) gives

$$\frac{de}{dt} = - \frac{\delta}{T_0}\left(2\pi a_0\right)^{\frac{1}{2}} \rho_p H_p^{\frac{1}{2}}\left(\frac{a_0}{a}\right)\frac{(1 + e)^{\frac{3}{2}}(1 - e)^{\frac{1}{2}}}{e^{\frac{1}{2}}}\left[1 + \frac{3}{8}\mu + 0\left(\frac{0.3}{z} , \frac{\mu}{z} , \frac{1}{z^2}\right)\right] . \tag{44}$$

To proceed further, $\rho_p H_p^{\frac{1}{2}}\left(a_0/a\right)$ must be expressed as a function of $e$ . On taking the reference distance $r_p$ in equation (3) as the initial value $r_{p0}$ , we may express $\rho_p$ in terms of $\rho_{p0}$ in the form

$$\rho_p = \rho_{p0}\left\{1 + b\sigma^2 H^{*2}\right\}\exp\left(\sigma H^*/H_{p0}\right) , \tag{45}$$

where $\sigma$ is given by (32). In the subsequent analysis the $0\left(\mu^2\right)$ terms will in the end be neglected; but there are many of them, and we need to evaluate them numerically to ensure that they do not accumulate to give an unacceptably large 0-term. The fuller equation for $b$ in terms of $\mu$ and $\sigma$ is therefore needed, and from (5) it is

$$b = \frac{\mu}{2H_p^2}\left\{1 - \mu\left(\sigma - \tfrac{1}{2}\sigma^2\right) + 0\left(\mu^2\right)\right\} . \tag{46}†$$

On substituting for $b$ from (46), eliminating $H^*/H_{p0}$ by means of (30) and expanding $\exp(1.4\mu\sigma)$ as a power series, we can rewrite equation (45) as

$$\rho_p = \rho_{p0}\left\{1 + \mu\sigma(1.4 + 0.5\sigma) + \mu^2\sigma^2\left(2.4 + 0.2\sigma + 0.25\sigma^2\right) + 0\left(\mu^3\right)\right\}\exp\sigma . \tag{47}$$

From (6) and (30) we find

$$H_p^{\frac{1}{2}} = H_{p0}^{\frac{1}{2}}\left\{1 - 0.5\mu\sigma - \mu^2\sigma(0.7 + 0.125\sigma) + 0\left(\mu^3\right)\right\} . \tag{48}$$

Multiplying (47) by (48) gives

$$\rho_p H_p^{\frac{1}{2}} = \rho_{p0}H_{p0}^{\frac{1}{2}}\left\{1 + \mu\sigma(0.9 + 0.5\sigma) + 0\left(0.1\mu^2\right)\right\}\exp\sigma , \tag{49}$$

or, on using (41) to evaluate $\exp\sigma$,

$$\rho_p H_p^{\frac{1}{2}} = \rho_{p0}H_{p0}^{\frac{1}{2}}\left\{\frac{e_0(1 + e)}{e\left(1 + e_0\right)}\right\}^{\frac{1}{2}}\left[1 + \mu\sigma\left\{1 + 0.5(\sigma - 0.2)\right\} + 0\left(\mu^2 , \frac{H}{4r_{p0}} , \frac{0.15}{z}\right)\right] . \tag{50}$$

Here $0 < \sigma < 0.5$, and so $|0.5\sigma(\sigma - 0.5)| < 0.075$, giving a term of $0(0.075\mu) = 0\left(0.375\mu^2\right)$ if $\mu = 0.2$ : this term can be absorbed in the $0\left(\mu^2\right)$ term in (50). Also, from (35)

$$\frac{a_0}{a} = \frac{1 - e}{1 - e_0} + 0\left(\frac{H}{2r_p}\right) . \tag{51}$$

On substituting (50), without the small term, and (51) into (44), and using (33) to approximate for $\sigma$, we obtain

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† This corrects an error in the formula for $b$ in Part IV, where the $\sigma$ and $\sigma^2$ are interchanged.

$$\frac{de}{dt} = -\frac{\rho_{p0}\delta}{T_0(1 - e_0)}\left(\frac{2\pi a_0 e_0 H_{p0}}{1 + e_0}\right)^{\frac{1}{2}} \frac{(1 + e)^2(1 - e)^{\frac{3}{2}}}{e}\left\{1 + \frac{3}{8}\mu + \mu\left(0.4 - 0.3e_0\right)\left(\frac{e_0}{e} - 1\right) + 0\left(\mu^2 , \frac{0.3}{z}\right)\right\} , \tag{52}$$

after dropping 0-terms that are smaller than others.

Equation (52) now has to be integrated. As in Part III we write

$$C = \frac{4\rho_{p0}\delta}{T_0\left(1 - e_0\right)}\left(\frac{2\pi a_0 e_0 H_{p0}}{1 + e_0}\right)^{\frac{1}{2}} , \tag{53}$$

so that (52) becomes

$$\frac{de}{dt} = -\frac{C}{4}\frac{(1 + e)^2(1 - e)^{\frac{3}{2}}}{e}\left\{1 + J\mu + \frac{K\mu}{e} + 0\left(\mu^2 , \frac{0.3}{z}\right)\right\} , \tag{54}$$

where $J$ and $K$ are constants given by

$$J = 0.3e_0 - 0.025 \tag{55}$$

$$K = \left(0.4 - 0.3e_0\right)e_0 . \tag{56}$$

Preparing for integration, we rewrite (54) as

$$-\frac{C}{4}\frac{dt}{de}(1 + J\mu)\left\{1 + 0\left(\mu^2 , \frac{0.3}{z}\right)\right\} = \frac{e}{(1 + e)^2(1 - e)^{\frac{3}{2}}} - \frac{K\mu}{(1 + e)^2(1 - e)^{\frac{3}{2}}} . \qquad \ldots\ldots (57)$$

On integration, (57) gives

$$Ct\left\{1 + J\mu\right\}\left\{1 + 0\left(\mu^2 , \frac{0.3}{z}\right)\right\} = \frac{3 + e_0}{\left(1 + e_0\right)\sqrt{\left(1 - e_0\right)}} - \frac{3 + e}{(1 + e)\sqrt{(1 - e)}} - \frac{1}{\sqrt{2}}\ln\frac{\left\{\sqrt{2} + \sqrt{\left(1 - e_0\right)}\right\}\sqrt{(1 + e)}}{\left\{\sqrt{2} + \sqrt{(1 - e)}\right\}\sqrt{\left(1 + e_0\right)}}$$

$$- K\mu\left[\frac{1 + 3e_0}{\left(1 + e_0\right)\sqrt{\left(1 - e_0\right)}} - \frac{1 + 3e}{(1 + e)\sqrt{(1 - e)}} + \frac{3}{2\sqrt{2}}\ln\left\{\frac{\sqrt{2} - \sqrt{\left(1 - e_0\right)}}{\sqrt{2} + \sqrt{\left(1 - e_0\right)}} \times \frac{\sqrt{2} + \sqrt{(1 - e)}}{\sqrt{2} - \sqrt{(1 - e)}}\right\}\right] . \qquad \ldots\ldots (58)$$

Equation (58) with $\mu = 0$ should be the same as equation (4.126) of Ref 11. Making the comparison, we see that the term $0(0.3/z)$ in (58), which is evaluated numerically in Part III, can be replaced by $0\left(H/5r_p\right) \simeq 0(0.002)$ .

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Equation (58) gives the variation of $e$ with $t$, but in order to bring it to the same format as in Part III, we need to define $t_L$, the lifetime of the satellite. In Part III, $t_L$ was defined as the value of $t$ for $e = 0$, even though the equation for $t$ in terms of $e$ was not valid for $e < 0.2$. This arbitrary definition was quite satisfactory for zero $\mu$, but causes difficulties when $\mu$ is non-zero because the $K\mu$ term in (57) becomes large relative to the 'main' term as $e \to 0$. As the definition of $t_L$ is arbitrary, and it needs to be approximately the satellite's lifetime, it should be defined in such a way as to be consistent with the theory for $e < 0.2$. This aim can be achieved by defining $t_L$ as the value of $t$ when $e = 0$ in equation (58) with the last of the three terms in $K\mu$ omitted. We then have

$$(1 + J\mu)Ct_L = \frac{3 + e_0}{\left(1 + e_0\right)\sqrt{\left(1 - e_0\right)}} - 3 - \frac{1}{\sqrt{2}} \ln \frac{\sqrt{2} + \sqrt{\left(1 - e_0\right)}}{\left(\sqrt{2} + 1\right)\sqrt{\left(1 + e_0\right)}}$$

$$- K\mu\left\{\frac{1 + 3e_0}{\left(1 + e_0\right)\sqrt{\left(1 - e_0\right)}} - 1\right\} . \tag{59}$$

Subtracting (58) from (59) gives:

$$(1 + J\mu)C\left[t_L - t\left\{1 + O\left(\mu^2, \frac{H}{5r_p}\right)\right\}\right] = f(e) - K\mu\left\{M(e) + L\left(e_0\right) - L(e)\right\} , \tag{60}$$

where
$$f(e) = \frac{3 + e}{(1 + e)\sqrt{(1 - e)}} - 3 - \frac{1}{\sqrt{2}} \ln \frac{\sqrt{2} + \sqrt{(1 - e)}}{(\sqrt{2} + 1)\sqrt{(1 - e)}} , \tag{61}$$

as in Part III, and $L(e)$ and $M(e)$ are new functions defined by

$$L(e) = \frac{3}{2\sqrt{2}} \ln \frac{\sqrt{2} + \sqrt{(1 - e)}}{\sqrt{2} - \sqrt{(1 - e)}} , \tag{62}$$

$$M(e) = \frac{1 + 3e}{(1 + e)\sqrt{(1 - e)}} - 1 . \tag{63}$$

As a guide to orders of magnitude, some values of $f(e)$, $L(e)$ and $M(e)$ are listed in Table 1.

Table 1

Values of $f(e)$, $L(e)$ and $M(e)$

| e | 0.2 | 0.4 | 0.6 | 0.8 |
|---|---|---|---|---|
| f(e) | 0.0775 | 0.3235 | 0.8405 | 2.112 |
| L(e) | 1.581 | 1.305 | 1.021 | 0.6946 |
| M(e) | 0.4907 | 1.029 | 1.767 | 3.224 |

Putting $e = e_0$ and $t = 0$ in (60) gives

$$(1 + J\mu)Ct_L = f(e_0) - K\mu M(e_0) \quad . \tag{64}$$

Dividing (60) by (64), we have

$$1 - \frac{t}{t_L}\left\{1 + 0\left(\mu^2 , \frac{H}{5r_p}\right)\right\} = \frac{f(e) - K\mu\left\{M(e) - L(e)\right\}}{f(e_0) - K\mu M(e_0)} - \frac{K\mu L(e_0)}{f(e_0) - K\mu M(e_0)} \quad . \tag{65}$$

Equation (65) gives $t/t_L$ in terms of $e$ and $e_0$, and Fig 3 shows how $e/e_0$ varies with $t/t_L$ for $e_0 = 0.4$, 0.6 and 0.8 with $\mu = 0$ and $\mu = 0.2$. The diagram shows that the effect of $\mu$ on $t/t_L$ is small and usually negligible. However, this does not mean that $\mu$ has a negligible effect on $e$ : equation (52) shows that $de/dt$ is increased, initially, by a factor $(1 + 3\mu/8)$ - that is 1.075 if $\mu = 0.2$. If this factor stayed constant, $t_L$ would be reduced by a factor 1.075 while the variation of $e$ with $t/t_L$ would be independent of $\mu$ ; in reality, however, the factor slowly increases as time goes on, thus producing the small variation of $e/e_0$ with $\mu$ seen in Fig 3.

## 8 LIFETIME IN TERMS OF THE INITIAL DECAY RATE

To obtain the lifetime $t_L$ in terms of $\dot{T}$ we go back to equation (25), but drop the $1/z$ term within curly brackets, which is of $0(H/2r_p)$, and the $\mu/z$ term, which is smaller. Then we have

$$\dot{T}_0 = \left(\frac{\Delta T}{T}\right)_0 = \left(\frac{3\Delta a}{2a}\right)_0 = -\frac{3}{2} a_0 \delta \rho_{p0}\left(\frac{2\pi H_{p0}}{a_0 e_0}\right)^{\frac{1}{2}} \frac{\left(1 + e_0\right)^{\frac{3}{2}}}{\left(1 - e_0\right)^{\frac{1}{2}}}\left\{1 + \frac{3}{8}\mu + 0\left(\frac{H}{2r_p}\right)\right\} \quad . \tag{66}$$

This may be written with the aid of (53) as

$$\dot{T}_0 = -\frac{3CT_0}{8e_0}\left(1 - e_0\right)^{\frac{1}{2}}\left(1 + e_0\right)^2\left\{1 + \frac{3}{8}\mu + 0\left(\frac{H}{2r_p}\right)\right\} \quad . \tag{67}$$

Multiplying (67) by (64), we find

$$t_L = -\frac{e_0 T_0}{\dot{T}_0} \frac{3\left(1 + e_0\right)^2\left(1 - e_0\right)^{\frac{1}{2}}}{8e_0^2} f(e_0)\left[1 - \mu\left\{J + \frac{KM(e_0)}{f(e_0)} - \frac{3}{8}\right\} + 0\left(\frac{H}{2r_p} , 0.2\mu^2\right)\right] \quad , \tag{68}$$

where $0.2\mu^2$ represents the maximum value of the neglected cross-multiplied $\mu$ terms. This is the required equation for the lifetime in terms of the initial decay rate.

We first need to check that it is consistent with the low-eccentricity theory. From the theory of Part IV, as revised in Ref 12, we find that, when $e_0 = 0.2$, the low-eccentricity thec·y gives

$$t_L = -\frac{3e_0T_0}{4\dot{T}_0} 1.25\left\{1 - 0.187\mu + 0\left(\mu^2\right)\right\} \qquad \text{(low-e theory)} . \tag{69}$$

When equation (68) is evaluated at $e_0 = 0.2$ , we find

$$t_L = -\frac{3e_0T_0}{4\dot{T}_0} 1.248\left\{1 - 0.091\mu + 0\left(\frac{H}{2r_p} , 0.2\mu^2\right)\right\} . \tag{70}$$

These differ by $0.096\mu = 0.5\mu^2$ if $\mu = 0.2$ and are therefore consistent. Thus equation (68) is acceptable even for $e_0 = 0.2$ , when the whole lifetime is outside the validity of the high-eccentricity theory. It is *a fortiori* acceptable for higher values of $e_0$ , because the added time from $e = e_0$ down to $e = 0.2$ will be given correctly by the high-e theory.

On evaluation, the term in curly brackets in (68) proves to have a value between 0.035 and 0.091 for $0.2 \leqslant e_0 \leqslant 0.8$ . Thus we can say that the constant-H lifetime formula, with $\mu = 0$ in (68), still applies, to $0\left(0.5\mu^2\right)$ .

## 9 DISCUSSION AND CONCLUSIONS

The analysis of section 6 shows that the constant-H theory for the variation of perigee height with eccentricity can still be used when H varies with height, if H is evaluated at a height 1.5 $H_p$ above the perigee height $y_p$ (for single-revolution results) or 1.4 $H_{p0}$ above initial perigee height $y_{p0}$ (for long-term results). With this refinement, Fig 3 of Part III, giving perigee height in terms of $e/e_0$ , and Fig 4 of Part III, giving $T/T_0$ against $\left(e_0 - e\right)$ , will remain valid. These results show that, with high-eccentricity orbits, simple exponential density models can still be successfully used if the choice of value of scale height is adjusted appropriately. In the analysis the value of $\mu (= dH/dy)$ is assumed to be less than 0.2, corresponding to perigee heights $y_p > 160$ km for low solar activity or $y_p > 190$ km for high solar activity.

The analysis of section 7 specifies the effect of non-zero $\mu$ on the variation of eccentricity with time. Equation (52) shows that initially the rate of decrease of e increases by a factor of $(1 + 0.375\mu)$. This factor becomes gradually larger as time goes on and at its maximum, for an orbit with $e_0 = 0.77$ and $e = 0.2$ , reaches $(1 + 0.857\mu)$. When time t is expressed as a fraction of the total lifetime $t_L$ , and e is plotted against $t/t_L$ , the effect of $\mu$ is generally negligible (see Fig 3). Consequently the variation of $T/T_0$ with $t/t_L$ is also virtually the same as in Fig 10 of Part III.

The curves of $e/e_0$ versus $t/t_L$ exhibit quite strong curvature, and an orbital parameter having a nearly linear variation with t would be welcome. As it happens, $z/z_0$ fulfils this requirement well, and Fig 4 shows how $z/z_0$ varies with $t/t_L$ .

The full formula for lifetime in terms of the initial decay rate is given by equation (68). On evaluating the $\mu$ terms in this equation, it is found that they are of order $0.5\mu^2$. Thus, although it is more accurate to use equation (68), the zero-$\mu$ equation for lifetime is essentially valid to order $\mu^2$.

In regard to past work the main implication of this Report is in confirming ideas, previously only surmised, (a) that $H$ should be evaluated at a height $\frac{3}{2}H$ above perigee in orbit determination, and (b) that, when scale height is determined from orbit analysis over a short time interval, this is the height at which $H$ applies.

The chief value of the theory for future work is in allowing more secure and effective use of simple atmospheric models in programs for orbit determination and analysis. The Report shows how best to interpret these models in order to achieve more accurate numerical results.

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## LIST OF SYMBOLS

| | |
|---|---|
| $a$ | semi major axis of orbit |
| $b$ | $= \frac{\mu}{2H_p^2}\left\{1 - \mu\left(\sigma - \frac{1}{2}\sigma^2\right) + O\left(\mu^2\right)\right\}$; see also equation (5) |
| $C$ | see equation (53) |
| $C_D$ | drag coefficient of satellite |
| $D$ | aerodynamic drag tangential to orbit |
| $e$ | eccentricity of orbit |
| $E$ | eccentric anomaly |
| $f(e)$ | see equation (61) |
| $F$ | $= \left(1 - \frac{r_{p0}w}{v_{p0}} \cos i\right)^2$ |
| $G_n$ | $= \int_0^\infty \lambda^{2n} \exp\left(-\lambda^2\right) d\lambda$ |
| $H$ | $= -\frac{\rho}{d\rho/dr}$ |
| $H_1$ | value of $H$ at a height $y_1$ |
| $H^*$ | $= H_{p0}(1 + 1.4\mu)$ |
| $i$ | inclination of orbit to equator |
| $J$ | $= 0.3e_0 - 0.25$ |
| $K$ | $= \left(0.4 - 0.3e_0\right)e_0$ |
| $L(e)$ | see equation (62) |
| $m$ | mass of satellite |
| $M(e)$ | see equation (63) |
| $Q$ | $= a(1 - e)$ |
| $r$ | distance from Earth's centre |
| $R$ | Earth's radius |
| $S$ | effective cross-sectional area of satellite |
| $t$ | time |
| $t_L$ | approximate lifetime after $e = e_0$ |
| $T$ | orbital period of satellite |
| $v$ | velocity of satellite relative to Earth's centre |
| $V$ | velocity of satellite relative to ambient air |

## LIST OF SYMBOLS (concluded)

| | | |
|---|---|---|
| $w$ | | angular velocity of atmosphere |
| $x$ | = | $ae$ |
| $y$ | | height above Earth's surface |
| $y_1$ | = | $y_p + \frac{3}{2}H_p$ |
| $z$ | = | $ae/H_p$ |
| $\alpha$ | = | $2bH_p^2$ |
| $\delta$ | = | $\dfrac{FSC_D}{m}$ |
| $\eta$ | = | $\left(r - r_p\right)/H_p$ |
| $\lambda$ | = | $\sqrt{\{z(1 - \cos E)\}}$ |
| $\mu$ | = | $dH/dy$ |
| $\pi$ | = | $3.14159 \ldots\ldots$ |
| $\rho$ | | air density |
| $\sigma$ | = | $\left(r_{p0} - r_p\right)/H^*$ |

### Suffixes

| | |
|---|---|
| 0 | initial value |
| p | value at perigee |

## REFERENCES


| No. | Author | Title, etc |
|---|---|---|
| 1 | D.G. King-Hele<br>G.E. Cook<br>D.M.C. Walker | The contraction of satellite orbits under the influence of air drag. Part I: With spherically symmetrical atmosphere.<br>RAE Technical Note GW 533 (1959)<br>[Also *Proc. Roy. Soc.* A, 257, 224-249 (1960)] |
| 2 | D.G. King-Hele<br>G.E. Cook<br>D.M.C. Walker | The contraction of satellite orbits under the influence of air drag. Part II: With oblate atmosphere.<br>RAE Technical Note GW 565 (1960)<br>[Also *Proc. Roy. Soc.* A, 264, 88-121 (1961)] |
| 3 | D.G. King-Hele | The contraction of satellite orbits under the influence of air drag. Part III: High-eccentricity orbits.<br>RAE Technical Note Space 1 (1962)<br>[Also *Proc. Roy. Soc.* A, 267, 541-557 (1962)] |
| 4 | D.G. King-Hele<br>G.E. Cook | The contraction of satellite orbits under the influence of air drag. Part IV: With scale height dependent on altitude.<br>RAE Technical Note Space 18 (1962)<br>[Also *Proc. Roy. Soc.* A, 275, 357-390 (1963)] |
| 5 | G.E. Cook<br>D.G. King-Hele | The contraction of satellite orbits under the influence of air drag. Part V: With day-to-night variation in air density.<br>*Phil. Trans. Roy. Soc.* A, 259, 33-67 (1965)<br>RAE Technical Report 64029 (1964) |
| 6 | G.E. Cook<br>D.G. King-Hele | The contraction of satellite orbits under the influence of air drag. Part VI: Near-circular orbits with day-to-night variation in air density.<br>*Proc. Roy. Soc. Lond.* A, 303, 17-35 (1968)<br>RAE Technical Report 67092 (1967) |
| 7 | G.G. Swinerd<br>W.J. Boulton | Contraction of satellite orbits in an oblate atmosphere with a diurnal density variation.<br>*Proc. Roy. Soc.* A, 383, 127-145 (1982) |
| 8 | G.G. Swinerd<br>W.J. Boulton | Near-circular satellite orbits in an oblate diurnally varying atmosphere.<br>*Proc. Roy. Soc.* A, 389, 153-170 (1983) |
| 9 | R.H. Gooding | The evaluation of the PROP 6 orbit determination program, and related topics.<br>RAE Technical Report 74164 (1974) |
| 10 | - | *CIRA 1972* (COSPAR International Reference Atmosphere 1972).<br>Berlin, Akademie-Verlag (1972) |

## REFERENCES (concluded)


| No. | Author | Title, etc |
|---|---|---|
| 11 | D.G. King-Hele | *Theory of satellite orbits in an atmosphere.* London, Butterworths (1964) |
| 12 | D.G. King-Hele | Methods for predicting satellite orbital lifetimes. *J. Brit. Interplan. Soc.*, 31, 181-196 (1978) RAE Technical Report 77111 (1977) |


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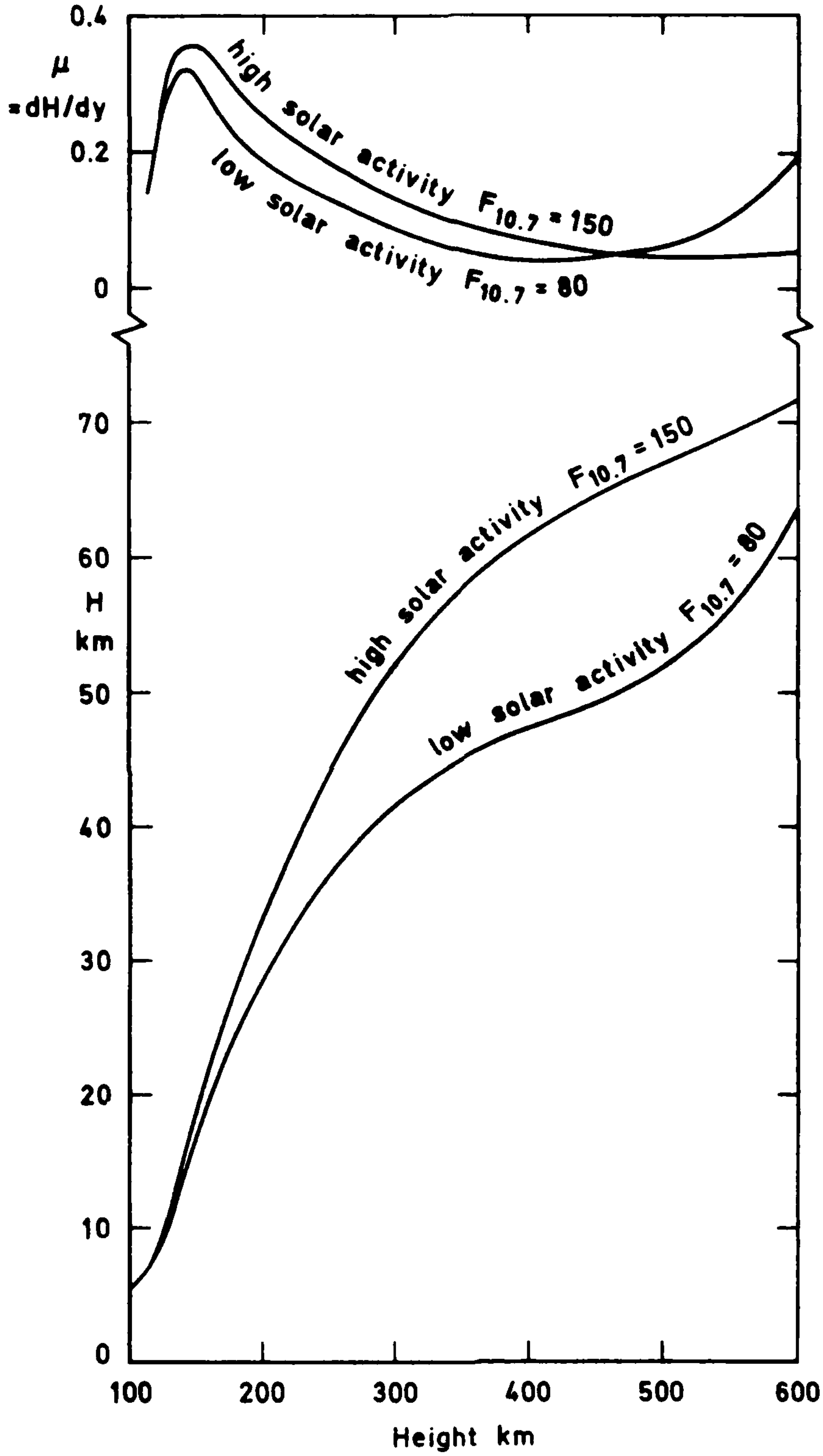


Fig 1 The variation with height y of density scale height H and its gradient $\mu = dH/dy$ as given by *CIRA 1972*, for high and low solar activity

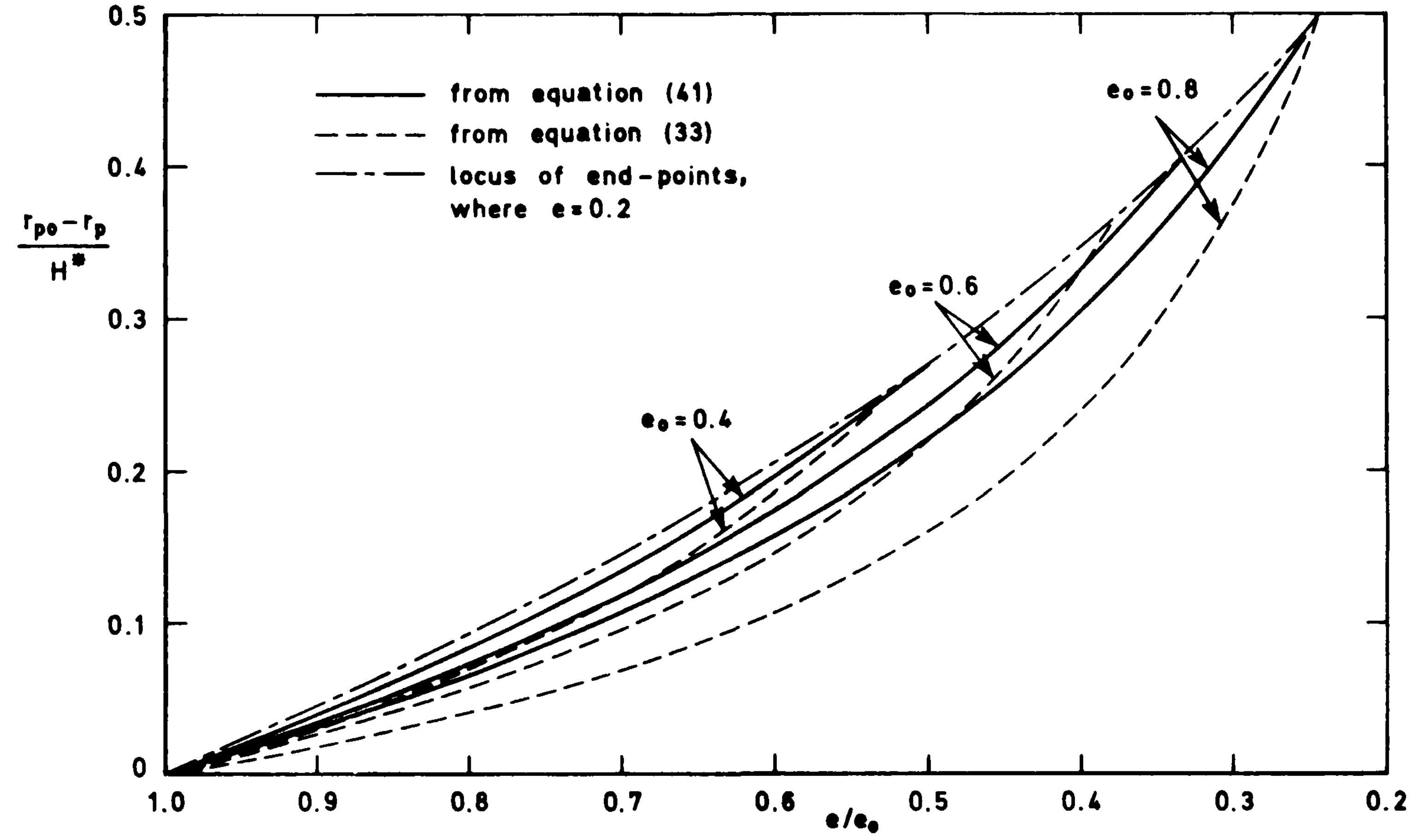


Fig 2 Variation of perigee distance with eccentricity, as given by equation (41) and the approximation (33)

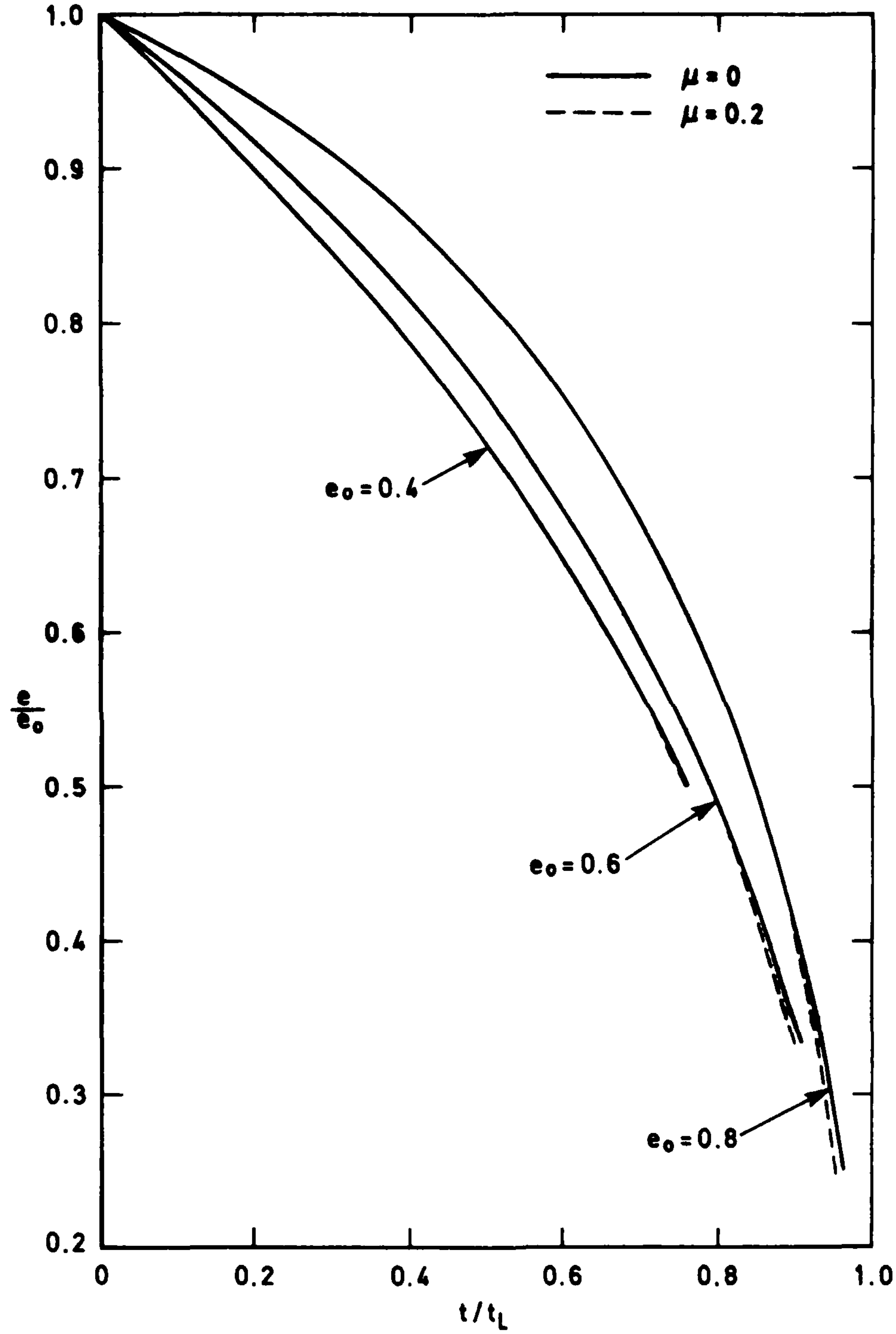


Fig 3 Variation of eccentricity with time

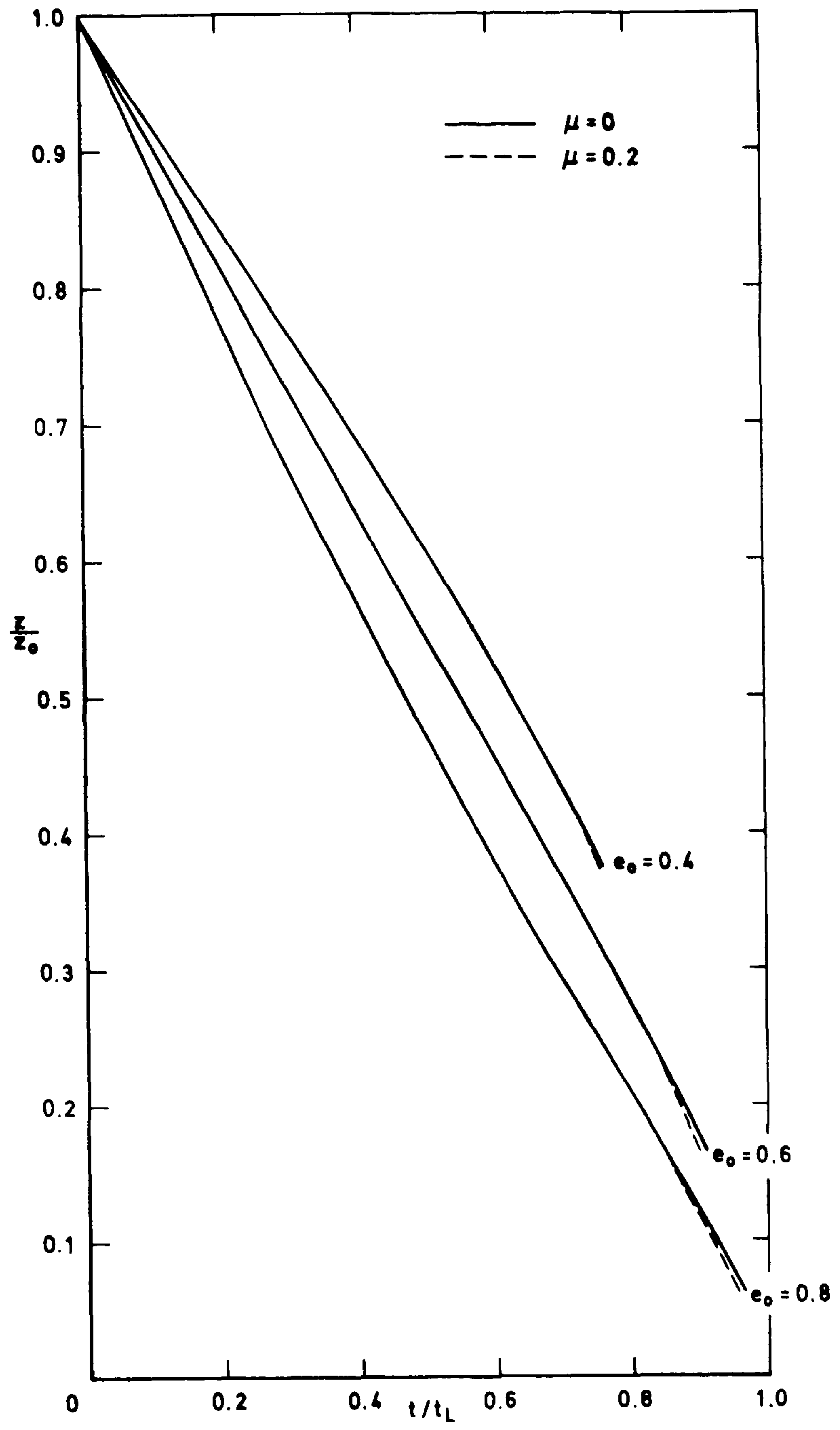


Fig 4 Variation of z with time

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